ably by decreasing the insertion loss of the output waveguide. This could be reduced by gold plating the inner surfaces or making the assembly from copper and by feeding the pump-power to the maser crystal in a more efficient way. However, it may also be possible to further improve the structure by i) increasing the length (now 5-cm active length). Compare the NRAO maser [1], which is 50 cm long; and/or ii) trying other cross sections, e.g., decreasing e further to get more tuning range (but less slowing) as indicated in Fig. 2(c).

If increasing the length of the maser is considered, it is necessary to carefully align the rutile crystal axes to within about $\pm 3°$ in order not to deteriorate the electronic gain by more than about 5 percent.

Since the maser is cooled by immersion in liquid helium, there is no need to take special precaution to have a good thermal contact between the rutile crystal and the metal structure. In a closed-cycle refrigerator, the situation is different and the teflon sheet should be replaced by a material with a high thermal conductivity such as sapphire.

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References

[1] C. R Moore, "A K-band maser with 500-MHz bandwidth," *IEEE Trans. Microwave Theory Tech* vol. MTT-28, pp. 149–151, 1980.

[2] K S. Yngvesson, A. C Cheung, M. F. Chui, A. G. Cardiasmenos, S. Y Wang, and C. H. Townes, "K-band traveling-wave maser using ruby," *IEEE Trans. Microwave Theory Tech.*, vol MTT-24, pp. 711–717, 1976

[3] E L. Kollberg and P T. Lewin, "Traveling-wave masers for radio astronomy in the frequency range 20–40 GHz," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 718–725, 1976.

[4] A. E. Siegman, *Microwave Solid-State Masers* New York: McGraw-Hill, 1964.

[5] T. Lewin, J. Järnmark, and E. Kollberg, "Theoretical and experimental evaluation of high perimittivity dielectric rod waveguides," *Int. J. Electron.*, vol. 49, pp. 147–159, 1980.

[6] J. Askne, E. Kollberg, and L. Pettersson, "Analysis of a waveguide partially filled with a dielectric material," in *Proc. 10th Euro. Microwave Conf.* (Warszawa), 1980.

[7] T. Lewin and E. Kollberg, "Rutile traveling wave masers for the frequency range 20–45 GHz," Res. Report 134, Research Laboratory of Electronics, Chalmers University of Technology, Göteburg, Sweden, 1978.

[8] J. Askne, E Kollberg, and L. Pettersson, "Propagation in a waveguide partially filled with an anisotropic dielectric material," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 795–799, May 1982.

Determination of Microwave Transistor Noise and Gain Parameters Through Noise-Figure Measurements Only

GIOVANNI MARTINES AND MARIO SANNINO

Abstract —**A novel method for measuring noise and gain parameters of linear two-ports solely from noise-figure measurements is applied here to perform noise and gain characterization of microwave transistors versus frequency and collector current in S-band.**

The method results in a simpler procedure and improved accuracy compared to conventional methods. In addition, a technique to estimate the loss of the input tuner of the measuring setup is presented, which yields a further improvement in accuracy.

As experimental verification, the noise and gain parameters of a microwave transistor versus collector current in the 2–4-GHz frequency range are reported.

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I. Introduction

At microwave frequencies it is convenient to write the two-port noise figure F in terms of input termination (or source) reflection coefficient $\Gamma_s = \rho_s \exp j\vartheta_s$ in the form [1], [2]

$$F(\Gamma_s) = F_0 + 4N_n \frac{|\Gamma_s - \Gamma_{0n}|^2}{(1-|\Gamma_s|^2)(1-|\Gamma_{0n}|^2)} \tag{1}$$

where F_0 (the minimum noise figure), $\Gamma_{0n} = \rho_{0n} \exp j\vartheta_{0n}$ (the optimum source reflection coefficient), and N_n (a terminal invariant parameter), are the four microwave noise parameters.

Similarly, we can introduce the following relationship for the available power gain:

$$\frac{1}{G_a(\Gamma_s)} = \frac{1}{G_{a0}} + 4N_g \frac{|\Gamma_s - \Gamma_{0g}|^2}{(1-|\Gamma_s|^2)(1-|\Gamma_{0g}|^2)} \tag{2}$$

where G_{a0}, $\Gamma_{0g} = \rho_{0g} \exp j\vartheta_{0g}$, and N_g are the four microwave gain parameters.

Based on standard methods of determining noise parameters established by the IRE [3], modernized computer-aided techniques have been developed [4], [5] and their drawbacks and improvements have been discussed [2], [6], [7].

The characterization of microwave transistors versus frequency and bias conditions in the S-band reported in this paper are obtained through a measuring technique which overcomes the usual time-consuming inconveniences.

Usually the device noise figure $F(\Gamma_s)$ is computed by means of the well-known Friis formula

$$F_m(\Gamma_s) = F(\Gamma_s) + \frac{F' - 1}{G_a(\Gamma_s)} \tag{3}$$

where F' and $F_m(\Gamma_s)$ represent the noise figure of all stages subsequent to the device and the overall measured noise figure, respectively.

From (3) it is obvious that the accuracy with which $G_a(\Gamma_s)$ is measured influences also the determination of $F(\Gamma_s)$ and consequently, of the device noise and gain parameters.

In the method proposed, both noise- and gain-parameter sets are simultaneously determined solely through noise-figure measurements. This simplifies the measurement procedure considerably and avoids separate instruments for the gain measurement. Further, since the device under test is driven at the noise level, nonlinearity effects are avoided; this improves the measurement accuracy by comparison with the methods which use signal generators to measure the gain. Commercial instruments for the simultaneous measurement of noise figure and gain of a device driven by a noise source are available at present (e.g., Hewlett-Packard mod. 8970 and Ailtech mod. 7380). These instruments, however, have been designed for measurements on the well-matched devices and, consequently, they are not convenient for transistor characterization. Furthermore, the method presented is believed to be more accurate in that the noise figure and gain of the device are determined through a fitting procedure of redundant experimental data.

It is noteworthy that for a correct measurement of $F_m(\Gamma_s)$, the loss of the tuner used as admittance transformer network must be measured. As pointed out recently, this measurement can be exactly performed through a complete characterization of the tuning network (e.g., in terms of its scattering parameters) [8]. The method used here, however, allows to estimate the above-mentioned losses through a proper processing technique of the data already derived from the parameter determination. Although from a theoretical view point this evaluation is approximate, the experimental results show that it is useful, provided that the tuner losses are not too large with respect to the noise figure.

The experimental verifications reported here regard the general case of the characterization of microwave transistors versus frequency and collector current.

Notes on the computer-aided data processing procedure and comments about the possibility of using the method in computer-controlled measuring systems are also reported.

II. Analysis of the Method

The theory of the method is based on the Friis formula (3) which, from (1) and (2), can be written

$$F_m(\Gamma_s) = F_0 + 4N_n \frac{|\Gamma_s - \Gamma_{0n}|^2}{(1-|\Gamma_s|^2)(1-|\Gamma_{0n}|^2)} + (F'-1)\left[\frac{1}{G_{a0}} + 4N_g \frac{|\Gamma_s - \Gamma_{0g}|^2}{(1-|\Gamma_s|^2)(1-|\Gamma_{0g}|^2)}\right]. \quad (4)$$

From (4) it appears that, in principle, measurements of F_m for four different values of Γ_s and for two different values of F' are sufficient to determine the eight unknown parameters avoiding thus an additional set of measurements specific for the determination of gain parameters. In order to smooth the experimental errors, however, redundant measurements of F_m for more than four values of Γ_s, and for more then two values of F' are necessary in practice [2], [4]–[6]. The novelty of this method lies in the use of different values of F' for a given set of Γ_s.

The measuring setup is shown in Fig. 1. It differs from a conventional noise-measuring system by the use of a step attenuator inserted between the stages subsequent to the device in order to easily obtain large variations in F'. The suggested measuring procedure is the following.

i) For a given frequency, measure the values of F' as defined by the Friis formula for each position of the attenuator. For unilateral devices the dependence of these values on Γ_s is negligible and this calibration procedure is performed one time for each frequency.

ii) Realize a value of Γ_s by adjusting the tuner and measure this value through the network analyzer; then measure $F_m(\Gamma_s)$ for each value of the attenuation.

iii) Repeat step ii) for other bias conditions of the device (if required).

iv) Repeat steps ii) and iii) for other values of Γ_s according to criteria previously suggested [2], [6].

When the device under test is matched (e.g., a low-noise amplifier), to implement the above procedure with a computer-controlled measuring system it is sufficient to use both bias supply and step attenuator in the programmable version. In the case of transistors, the full automation may prove difficult because step ii) would require the adoption of particular stripline tuners [5] which, in general, are characterized by large losses as compared to the conventional (double-stub or slide-screw) waveguide or coaxial tuners.

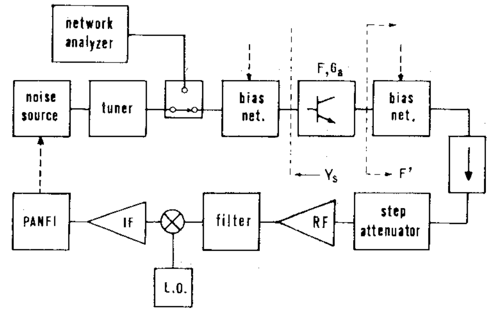

Fig. 1. Noise- and gain-parameter measuring system.

III. Computer-Aided Parameter Evaluation

The computation procedure for the determination of the device noise and gain parameters requires the solution of an 8×8 equation system derived from (4) with redundant experimental data.

For computational convenience, however, we can write (4) in the linearized form

$$F(\Gamma_s) = a_n + b_n \frac{1}{1-\rho_s^2} + c_n \frac{\cos\vartheta_s}{1-\rho_s^2} + d_n \frac{\sin\vartheta_s}{1-\rho_s^2} \quad (5)$$

where a_n, b_n, c_n and d_n are *indirect* noise parameters (similar to those defined by Fukui) related to the conventional ones F_0, ρ_{0n}, ϑ_{0n}, and N_n through known relationships [2]. Similarly, (2) is written in the form

$$\frac{1}{G_a(\Gamma_s)} = a_g + b_g \frac{1}{1+\rho_s^2} + c_g \frac{\cos\vartheta_s}{1-\rho_s^2} + d_g \frac{\sin\vartheta_s}{1-\rho_s^2} \quad (6)$$

where a_g, b_g, c_g, and d_g are the *indirect* gain parameters given by

$$\frac{1}{G_{a0}} = a_g \frac{b_g + \Delta_g}{2}$$

$$N_g = \frac{\Delta_g}{4}; \; \rho_{0g} = \left(\frac{b_g - \Delta_g}{b_g + \Delta_g}\right)$$

$$\vartheta_{0g} = \tan^{-1}\frac{d_g}{c_g}$$

with

$$\Delta_g = \left(b_g^2 - c_g^2 - d_g^2\right)^{1/2}. \quad (7)$$

Further, using (5) and (6), we can write the measured noise figure (3) as

$$F_m(\Gamma_s) = a_m + b_m \frac{1}{1+\rho_s^2} + c_m \frac{\cos\vartheta_s}{1+\rho_s^2} + d_m \frac{\sin\vartheta_s}{1+\rho_s^2}. \quad (8)$$

The new set of parameters

$$a_m = a_n + (F'-1)a_g \qquad b_m = b_n + (F'-1)b_g$$
$$c_m = c_n + (F'-1)c_g \qquad d_m = d_n + (F'-1)d_g \quad (9)$$

can be regarded as the indirect noise parameters of the measuring setup for a given value of F'. After a_m, b_m, c_m, and d_m are determined from (8), a_n, b_n, c_n, and d_n and a_g, b_g, c_g, and d_g are determined from (9).

From the above, it appears that the method yields also an original procedure to examine the measurement accuracy. To this end, the experimenter must determine experimentally the optimum values F_{m0}, ρ_{m0}, ϑ_{m0} of the measuring system noise parameters for each value of F' by adjusting the input tuner. These values are then compared to the ones computed by means of (9).

Comparison between measured and computed values allows the estimation of accuracy.

In a similar way, it is possible to perform a simple evaluation of the loss of the tuner used as admittance transformer network, which influences to the same extent (in decibels) both device noise figure and gain. To this end, the values F_{50} and G_{50} of F and G under input matched conditions (usually 50Ω) are compared through the device noise and gain parameters; they are then compared with their values as measured by connecting to the device under test the matched noise source directly, i.e., without the input tuner.

The method presented here can be also applied when noise-parameter measurements are performed through the two-channel measuring system (i.e., without filtering of the image frequency arising from frequency conversion) which has been previously presented from both theoretical [9] and experimental [10] viewpoints.

IV. Experimental Verifications

After the effectiveness of the method has been examined extensively through computer-aided simulation, experiments have been carried out using some low-noise transistors.

Noise-figure measurements have been performed through the setup of Fig. 1 for ten different values of Γ_s and four different values of the attenuation (namely 0, 3, 6, and 10 dB). The values of the attenuation depend on the gain of the device under test. High values of the device gain require high values of the attenuation in order to obtain noticeable increments of the meter reading F_m, necessary for high accuracy. The solid-state noise generator has an excess noise ratio of 15.1 dB in S-band and is switched ON and OFF by the precision automatic noise-figure indicator (PANFI). The values of Γ_s are realized through a low-loss double-stub tuner and are measured on-line by means of the automatic network analyzer connected to the tuner output through a microwave switch. This permits also the determination of the optimum noise conditions F_{m0}, Γ_{m0} for each value of F'. Obviously, in order to improve the accuracy, the values of Γ_s are chosen in a region of the Smith chart close to Γ_{m0}. The noise and gain parameters have been derived by processing the experimental data with an HP 9830 desk computer using the IBM subroutine SIM Q (translated into BASIC language) for solving systems of linear equations. Since F_0, ρ_{0n}, N_n, and G_{a0}, ρ_{0g}, N_g are terminal invariants under lossless transformation [2], ϑ_{0n} and ϑ_{0g} only are corrected taking into account the electrical length which separates the Γ_s measuring plane and the actual transistor input.

The characterization of a bipolar transistor (AT-4642; Avantek; common emitter configuration; $V_{CE}=10$ V) in terms of noise and gain parameters in the frequency range 2–4 GHz for two values of collector current is shown in Fig. 2. In Fig. 3 the same parameters are shown versus collector current for two different values of frequency. The computed values F_G (associated noise figure) corresponding to Γ_{0g} and G_F (associated gain) corresponding to Γ_{0n}, are reported in Fig. 4 versus collector current. The values of F_{50} and G_{50} under input matched conditions as computed through the noise and gain parameters are shown in Fig. 4 together with the measured data. These parameters are also shown versus frequency in Fig. 5(a) for the optimum value of collector current from the noise point-of-view, as derived from Fig. 3 (which coincides with the value suggested by the transistor manufacturer). As stated above, comparison between computed and measured values yields the loss of the input tuner. From the figures, it appears that this loss is, obviously, independent of collector current and increases as the frequency increases, beginning not negligible ($\geqslant 0.1$ dB) for frequencies higher than 2.5 GHz, as shown in Fig. 5(b). These considerations on the tuner loss are to be taken into account in reading the values of noise figure (subtract the loss in decibels) and gain (add the loss in decibels) shown in Fig. 2.

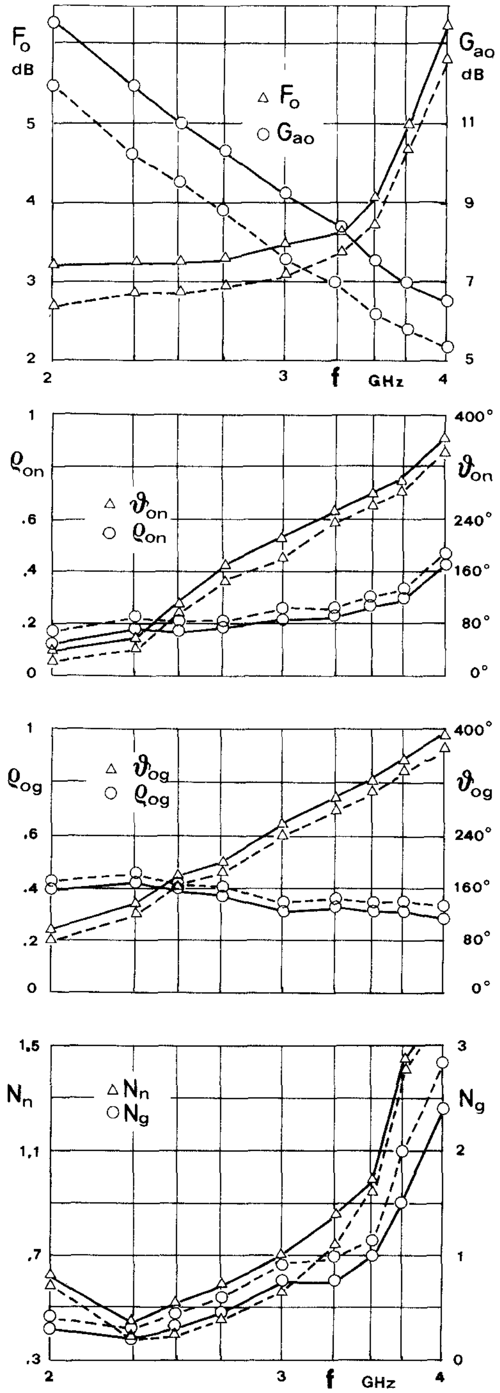

Fig 2. Microwave noise and gain parameters versus frequency for two different values of collector current (---5 mA, —15 mA), and for a transistor AT 4642 (Avantek; common emitter configuration; $V_{CE}=10$ V).

V. Conclusion

A method for the simultaneous determination of device noise and gain parameters through noise-figure measurements only has been presented, which offers some advantages as compared to the conventional methods which require two different measurement

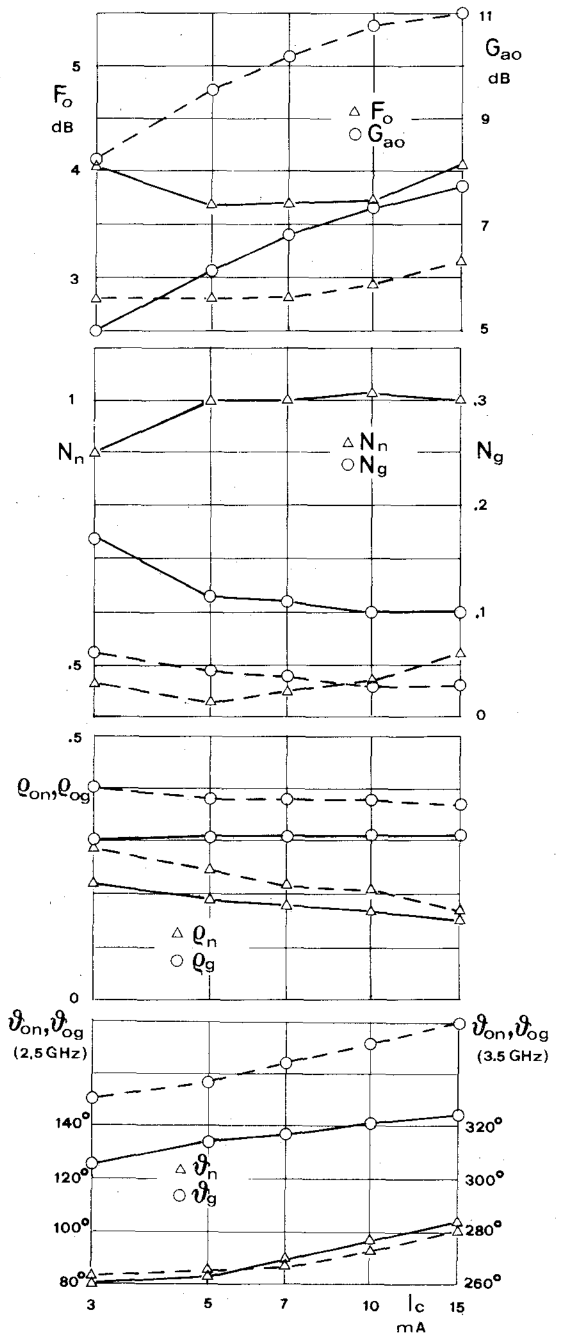

Fig. 3. Microwave noise and gain parameters versus collector current for two different values of frequency (---2.5 GHz, —3.5 GHz) for the transistor of Fig. 2.

procedures for the determination of the two parameter sets.

The method employs a conventional noise-figure measuring setup; the only difference is a step attenuator inserted after the device under test so that it influences the noise meter reading depending on the device gain.

By means of a proper computer-aided data processing technique, the eight parameters are determined, the accuracy of the measurements is examined, and evaluation of the input tuner loss is performed.

The method is very useful because it does not require specific instrumentation for gain measurements. The main advantage is the drastic reduction of measurement time.

ACKNOWLEDGMENT

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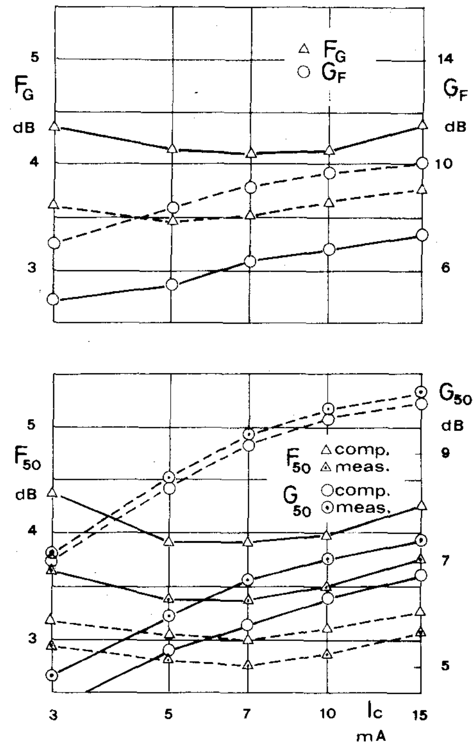

Fig. 4. Values of F_G and G_F of noise figure and gain, measured and computed values of F_{50} and G_{50}, versus collector current at two different values of frequency (---2.5 GHz, —3.5 GHz) for the transistor of Fig. 2.

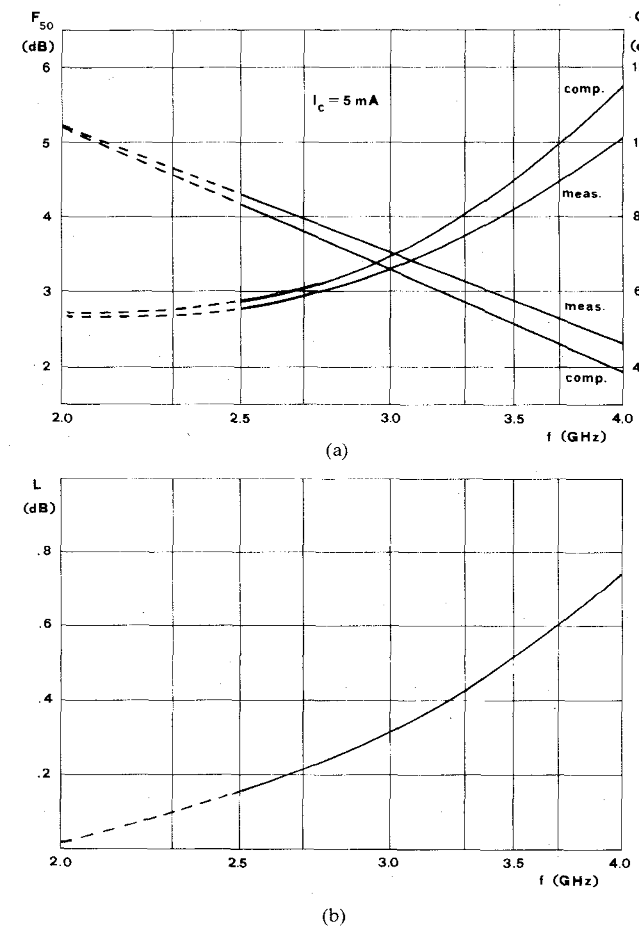

Fig. 5. Computed and measured values of F_{50} and G_{50} versus frequency, (a) for the transistor of Fig. 2 at $I_c = 5$ mA, and (b) loss of the input tuner of the measuring system as derived from (a).

References[1]

[1] H. Fukui, "Available power gain, noise figure, and noise measure of twoport and their graphical representations," *IEEE Trans. Circuit Theory*, vol. CT-15, pp 137–142, June 1966.

[2] G. Caruso and M. Sannino, "Computer-aided determination of microwave twoport noise parameters," *IEEE Trans. Microwave Theory Tech.*, vol MTT-26, pp. 639–642, Sept. 1978.

[3] IRE Subcommittee on Noise, "IRE Standards on methods of measuring noise in linear two-ports, 1959," *Proc. IRE*, vol 48, Jan. 1960, pp. 60–68.

[4] R Q. Lane, "The determination of device noise parameters," *Proc. IEEE*, vol. 57, Aug. 1969, pp. 1461–1462.

[5] R. Q. Lane, "A microwave noise and gain parameter test set," in *1978 IEEE ISSCC Dig. Tech Papers*, Feb 1978, pp. 172–173, and 274.

[6] M. Sannino, "On the determination of device noise and gain parameters," *Proc. IEEE*, vol. 67, Sept. 1979, pp. 1364–1366

[7] M Mitama and H. Katoh, "An improved computational method for noise parameter measurements," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 612–615, June 1979.

[8] E. W Strid, "Measurements of losses in noise matching networks," *IEEE Trans. Microwave Theory Tech*, vol. MTT-29, pp. 247–253, Mar 1981

[9] G. Caruso and M. Sannino, "Analysis of frequency-conversion techniques in measurements of microwave transistor noise temperatures," *IEEE Trans. Microwave Theory Tech.*, vol MTT-25, pp. 870–873, Nov. 1977.

[10] G. Caruso and M. Sannino, "Determination of microwave two-port noise parameters through computer-aided frequency conversion techniques," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp 779–782, Sept. 1979.

[1]References [1], [3]–[8] are also in the IEEE book "*Low Noise Microwave Transistors and Amplifiers*" H. Fukui, Ed, 1981.

Comparison Decrement Method for Microwave Resonator Q Measurements

IVAN KNEPPO

***Abstract*— The sensitivity of resonator quality-factor measurements using the decrement method can be improved by the comparison of the decay curve of a measured resonator with that of a reference standard resonator. The decay curves are compared using the integral of the function given as difference between decay curves. The comparison method, to be proposed, also has the advantage that the calibration of neither time base nor power scale is needed. Some results of the experimental verification are presented.**

I. Introduction

The decrement method of measuring Q of a microwave resonator is a well-known and useful measuring method, especially if the resonator quality is high. Accuracy can be quite high, 0.5 percent [1]. However, in such a case when a small change of Q ought to be measured, e.g., cavity perturbation measurements of small loss tangents, or loss tangents smaller than the uncertainty due to the inaccuracy of the measurement method, the obtained results are neither reproducible nor satisfactory.

One way to enhance the sensitivity of the decrement-measuring set to the change of measured variables is to compare the decay curve of the measured resonator with that of a calibrated reference standard resonator.

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II. Theory

The method, to be proposed, is based on processing the difference between the square-law detector output voltages of the reference resonator

$$u_R(t) = \beta_R P_{OR} \exp(-\omega_{OR} t / Q_{LR}) \tag{1}$$

and that of the measured resonator

$$u_M(t) = \beta_M P_{OM} \exp(-\omega_{0M} t / Q_{LM}) \tag{2}$$

i.e.,

$$u_D(t) = u_M(t) - u_R(t). \tag{3}$$

If both microwave detectors are paired $\beta_M = \beta_R$, and the initial values of the resonator's output power and resonant frequencies are equal, $P_{OM} = P_{OR}$ and $\omega_{OM} = \omega_{OR}$, respectively, then, consequently, $u_D(t)$ has the form

$$u_D(t) = \beta_R P_{OR} [\exp(-\omega_{OR} t / Q_{LM}) - \exp(-\omega_{OR} t / Q_{LR})] \tag{4}$$

where Q_{LM} and Q_{LR} are the loaded Q's of the measured and reference resonator, respectively. Integrating $u_D(t)$ and $u_R(t)$ over interval $\langle 0, \infty)$ we obtain

$$\iota_D = \int_0^\infty u_D(t)\, dt = \beta_R P_{OR} (Q_{LM} - Q_{LR}) / \omega_{OR} \tag{5}$$

$$\iota_R = \int_0^\infty u_R(t)\, dt = \beta_R P_{OR} Q_{LR} / \omega_{OR}. \tag{6}$$

Dividing (5) by (6) we obtain the final expression

$$\iota_D / \iota_R = (Q_{LM} - Q_{LR}) / Q_{LR} \tag{7}$$

which is very convenient for computing the unknown Q_{LM} on the basis of measured ratio ι_D / ι_R and previously known Q_{LR}.

III. Procedure

The arrangement for measuring Q by comparison of decay curves is shown in Fig. 1. Microwave power from the pulse modulated source is divided into two equal portions, one for measured and one for reference resonator energization. Output voltages from detectors are fed to the inputs of the differential preamplifier of the sampling oscilloscope. The measurement is initiated with tuning the reference resonator onto the frequency of the microwave generator output, and recording the resonator decay power curve $u_R(t)$. The variable attenuator inserted into reference tract and the differential preamplifier gain are set in such a manner that the record optimally covers the whole area of the CRT screen and the value A_1 of the preamplifier gain is recorded. Secondly, the measured resonator is also tuned to the generator wave frequency as indicated by the lowering of the transient response, and the attenuator in the measuring tract is set to zeroing the transient response at $t = 0$. The final $u_D(t)$ is recorded and the gain of the differential preamplifier should be increased to the value A_2 in order to obtain a good resolvable record again. The measurement process is completed by the measurement of the area under $u_R(t)$ and $u_D(t)$ with the use of a polar planimeter and computing Q_{LM} according to the formula

$$Q_{LM} = Q_{LR} [1 + \iota_D / (K \iota_R)] \tag{8}$$

where $K = A_2 / A_1$. In practice, we integrate over a finite interval instead of the theoretically assumed $\langle 0, \infty)$, requiring that the